Proportional Relationships

MODULE 3

COMMON CORE

ESSENTIAL QUESTION

How can you use proportional relationships to solve real-world problems?

Real-World Video

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Speedboats can travel at fast rates while sailboats travel more slowly. If you graphed distance versus time for both types of boats, you could tell by the steepness of the graph which boat was faster.

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Complete these exercises to review skills you will need for this module.

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Write Fractions as Decimals

EXAMPLE $\frac{1.7}{2.5} = ?$

Multiply the numerator and the denominator by a power of 10 so that the denominator is a whole number.

$$\frac{1.7 \times 10}{2.5 \times 10} = \frac{17}{25}$$

Write the fraction as a division problem.
Write a decimal point and zeros in the dividend.
Place a decimal point in the quotient.
Divide as with whole numbers.

$$\begin{array}{r} 0.68 \\ 25\overline{)17.00} \\ -15\,0 \\ \hline 2\,00 \\ -2\,00 \\ \hline 0 \end{array}$$

Write each fraction as a decimal.

1. $\frac{3}{8}$ ________ **2.** $\frac{0.3}{0.4}$ ________ **3.** $\frac{0.13}{0.2}$ ________

4. $\frac{0.39}{0.75}$ ________ **5.** $\frac{4}{5}$ ________ **6.** $\frac{0.1}{2}$ ________

7. $\frac{3.5}{14}$ ________ **8.** $\frac{7}{14}$ ________ **9.** $\frac{0.3}{10}$ ________

Solve Proportions

EXAMPLE $\frac{5}{7} = \frac{x}{14}$

$\frac{5 \times 2}{7 \times 2} = \frac{x}{14}$ $7 \times 2 = 14$, so multiply the numerator and denominator by 2.

$\frac{10}{14} = \frac{x}{14}$ $5 \times 2 = 10$

$x = 10$

Solve each proportion for *x*.

10. $\frac{20}{18} = \frac{10}{x}$ ________ **11.** $\frac{x}{12} = \frac{30}{72}$ ________ **12.** $\frac{x}{4} = \frac{4}{16}$ ________

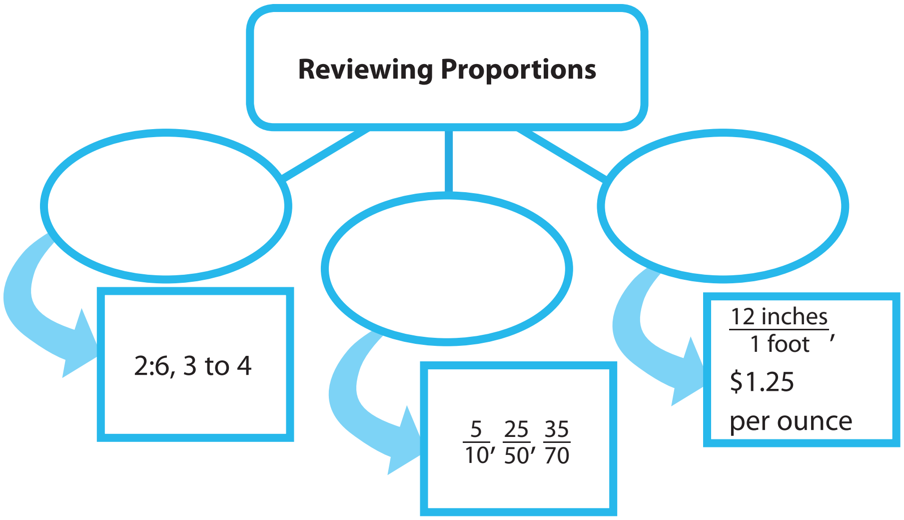
13. $\frac{11}{x} = \frac{132}{120}$ ________ **14.** $\frac{36}{48} = \frac{x}{4}$ ________ **15.** $\frac{x}{9} = \frac{21}{27}$ ________

16. $\frac{24}{16} = \frac{x}{2}$ ________ **17.** $\frac{30}{15} = \frac{6}{x}$ ________ **18.** $\frac{3}{x} = \frac{18}{36}$ ________

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the diagram.



Vocabulary

Review Words

- constant *(constante)*
- ✔ equivalent ratios *(razones equivalentes)*
- proportion *(proporción)*
- rate *(tasa)*
- ✔ ratios *(razón)*
- ✔ unit rates *(tasas unitarias)*

Preview Words

- constant of proportionality *(constante de proporcionalidad)*
- proportional relationship *(relación proporcional)*
- rate of change *(tasa de cambio)*
- slope *(pendiente)*

Understand Vocabulary

Match the term on the left to the definition on the right.

1. unit rate	A. A constant ratio of two variables related proportionally.
2. constant of proportionality	B. A rate in which the second quantity in the comparison is one unit.
3. proportional relationship	C. A relationship between two quantities in which the ratio of one quantity to the other quantity is constant.

Active Reading

Key-Term Fold Before beginning the module, create a key-term fold to help you learn the vocabulary in this module. Write the highlighted vocabulary words on one side of the flap. Write the definition for each word on the other side of the flap. Use the key-term fold to quiz yourself on the definitions used in this module.

MODULE 3

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

COMMON CORE 8.EE.5

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Key Vocabulary

proportional relationship *(relación proporcional)*
A relationship between two quantities in which the ratio of one quantity to the other quantity is constant.

slope *(pendiente)*
A measure of the steepness of a line on a graph; the rise divided by the run.

unit rate *(tasa unitaria)*
A rate in which the second quantity in the comparison is one unit.

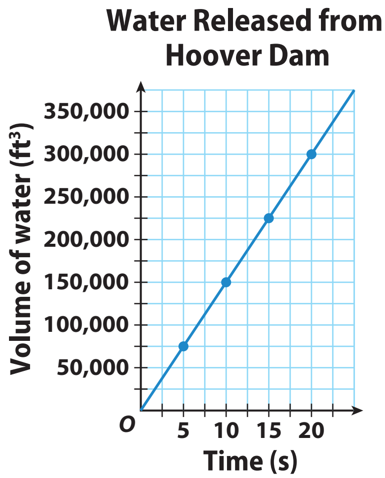
What It Means to You

You will use data from a table and a graph to apply your understanding of rates to analyzing real-world situations.

UNPACKING EXAMPLE 8.EE.5

The table shows the volume of water released by Hoover Dam over a certain period of time. Use the data to make a graph. Find the slope of the line and explain what it shows.

Water Released from Hoover Dam	
Time (s)	Volume of water (ft^3)
5	75,000
10	150,000
15	225,000
20	300,000



The slope of the line is 15,000. This means that for every second that passed, 15,000 ft^3 of water was released from Hoover Dam.

Suppose another dam releases water over the same period of time at a rate of 180,000 ft^3 per minute. How do the two rates compare?

180,000 ft^3 per minute is equal to 3,000 ft^3 per second. This rate is one fifth the rate released by the Hoover Dam over the same time period.

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LESSON 3.1

Representing Proportional Relationships

COMMON CORE 8.EE.6

...derive the equation $y = mx$ for a line through the origin... *Also 8.F.4*

ESSENTIAL QUESTION

How can you use tables, graphs, and equations to represent proportional situations?

EXPLORE ACTIVITY Real World

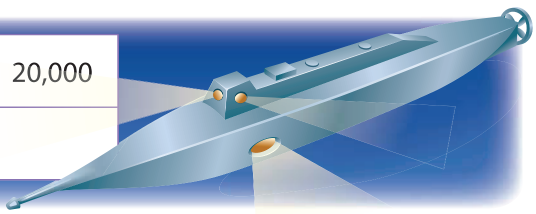
COMMON CORE **Prep for 8.EE.6**

Representing Proportional Relationships with Tables

In 1870, the French writer Jules Verne published *20,000 Leagues Under the Sea*, one of the most popular science fiction novels ever written. One definition of a *league* is a unit of measure equaling 3 miles.

A Complete the table.

Distance (leagues)	1	2	6		20,000
Distance (miles)	3			36	



B What relationships do you see among the numbers in the table?

____

____

C For each column of the table, find the ratio of the distance in miles to the distance in leagues. Write each ratio in simplest form.

$\frac{3}{1} = \square$ $\quad \frac{\square}{2} = \square$ $\quad \frac{\square}{6} = \square$ $\quad \frac{36}{\square} = \square$ $\quad \frac{\square}{20{,}000} = \square$

D What do you notice about the ratios? ______________________

Reflect

1. If you know the distance between two points in leagues, how can you find the distance in miles? ______________________

2. If you know the distance between two points in miles, how can you find the distance in leagues? ______________________

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Representing Proportional Relationships with Equations

The ratio of the distance in miles to the distance in leagues is constant. This relationship is said to be *proportional*. A **proportional relationship** is a relationship between two quantities in which the ratio of one quantity to the other quantity is constant.

A proportional relationship can be described by an equation of the form $y = kx$, where k is a number called the **constant of proportionality**.

Sometimes it is useful to use another form of the equation, $k = \frac{y}{x}$.

EXAMPLE 1 Real World

COMMON CORE 8.EE.6

Meghan earns $12 an hour at her part-time job. Show that the relationship between the amount she earned and the number of hours she worked is a proportional relationship. Then write an equation for the relationship.

STEP 1 Make a table relating amount earned to number of hours.

For every hour Meghan works, she earns $12. So, for 8 hours of work, she earns $8 \times \$12 = \$96$.

Number of hours	1	2	4	8
Amount earned ($)	12	24	48	96

STEP 2 For each number of hours, write the relationship of the amount earned and the number of hours as a ratio in simplest form.

$\frac{\text{amount earned}}{\text{number of hours}}$ $\quad \frac{12}{1} = \frac{12}{1} \quad \frac{24}{2} = \frac{12}{1} \quad \frac{48}{4} = \frac{12}{1} \quad \frac{96}{8} = \frac{12}{1}$

Since the ratios for the two quantities are all equal to $\frac{12}{1}$, the relationship is proportional.

STEP 3 Write an equation.

First tell what the variables represent.

Let x represent the number of hours.
Let y represent the amount earned.

Use the ratio as the constant of proportionality in the equation $y = kx$.

The equation is $y = \frac{12}{1}x$ or $y = 12x$.

Math Talk
Mathematical Practices
Describe two real-world quantities with a proportional relationship that can be described by the equation $y = 25x$.

YOUR TURN

3. Fifteen bicycles are produced each hour at the Speedy Bike Works. Show that the relationship between the number of bikes produced and the number of hours is a proportional relationship. Then write an equation for the relationship. ____________

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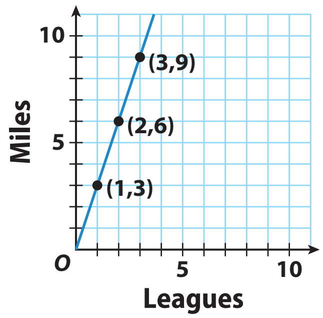
Representing Proportional Relationships with Graphs

You can represent a proportional relationship with a graph. The graph will be a line that passes through the origin (0, 0). The graph shows the relationship between distance measured in miles to distance measured in leagues.



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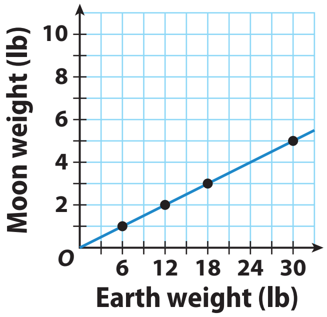
EXAMPLE 2 Real World

COMMON CORE 8.EE.6

The graph shows the relationship between the weight of an object on the Moon and its weight on Earth. Write an equation for this relationship.



STEP 1 Use the points on the graph to make a table.

Earth weight (lb)	6	12	18	30
Moon weight (lb)	1	2	3	5

STEP 2 Find the constant of proportionality.

$\frac{\text{Moon weight}}{\text{Earth weight}}$ $\quad \frac{1}{6} = \frac{1}{6} \quad \frac{2}{12} = \frac{1}{6} \quad \frac{3}{18} = \frac{1}{6} \quad \frac{5}{30} = \frac{1}{6}$

The constant of proportionality is $\frac{1}{6}$.

STEP 3 Write an equation.

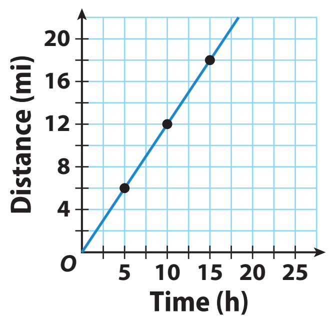
Let x represent weight on Earth.

Let y represent weight on the Moon.

The equation is $y = \frac{1}{6}x$. *Replace k with $\frac{1}{6}$ in $y = kx$.*

YOUR TURN

The graph shows the relationship between the amount of time that a backpacker hikes and the distance traveled.



4. What does the point (5, 6) represent?

____

5. What is the equation of the relationship?

____

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Guided Practice

1. **Vocabulary** A proportional relationship is a relationship between two quantities in which the ratio of one quantity to the other quantity

is / is not constant.

2. **Vocabulary** When writing an equation of a proportional relationship in the form $y = kx$, k represents the ______________________________.

3. Write an equation that describes the proportional relationship between the number of days and the number of weeks in a given length of time. (Explore Activity and Example 1)

a. Complete the table.

Time (weeks)	1	2	4		10
Time (days)	7			56	

b. Let x represent ______________________________.

Let y represent ______________________________.

The equation that describes the relationship is ________________.

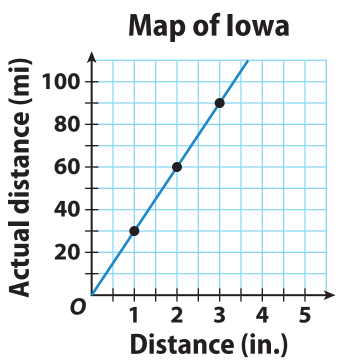
Each table or graph represents a proportional relationship. Write an equation that describes the relationship. (Example 1 and Example 2)

4. **Physical Science** The relationship between the numbers of oxygen atoms and hydrogen atoms in water

Oxygen atoms	2	5		120
Hydrogen atoms	4		34	

____

5.



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ESSENTIAL QUESTION CHECK-IN

6. If you know the equation of a proportional relationship, how can you draw the graph of the equation?

____

____

Name ____________ Class ____________ Date ____________

3.1 Independent Practice

COMMON CORE 8.EE.6, 8.F.4

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The table shows the relationship between temperatures measured on the Celsius and Fahrenheit scales.

Celsius temperature	0	10	20	30	40	50
Fahrenheit temperature	32	50	68	86	104	122

7. Is the relationship between the temperature scales proportional? Why or why not?

____

____

8. Describe the graph of the Celsius-Fahrenheit relationship.

____

____

9. **Analyze Relationships** Ralph opened a savings account with a deposit of \$100. Every month after that, he deposited \$20 more.

a. Why is the relationship described not proportional?

____

____

b. How could the situation be changed to make the situation proportional?

____

____

10. **Represent Real-World Problems** Describe a real-world situation that can be modeled by the equation $y = \frac{1}{20}x$. Be sure to describe what each variable represents.

____

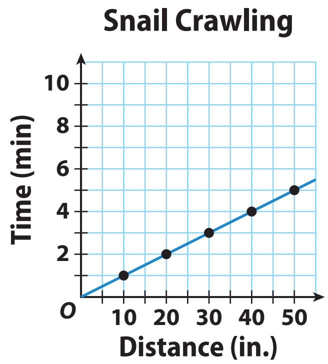
____

Look for a Pattern **The variables x and y are related proportionally.**

11. When $x = 8$, $y = 20$. Find y when $x = 42$. ______________

12. When $x = 12$, $y = 8$. Find x when $y = 12$. ______________

13. The graph shows the relationship between the distance that a snail crawls and the time that it crawls.



a. Use the points on the graph to make a table.

Distance (in.)					
Time (min)					

b. Write the equation for the relationship and tell what each variable represents.

____

____

c. How long does it take the snail to crawl 85 inches? ____________

Work Area

H.O.T. FOCUS ON HIGHER ORDER THINKING

14. Communicate Mathematical Ideas Explain why all of the graphs in this lesson show the first quadrant but omit the other three quadrants.

____

____

____

____

15. Analyze Relationships Complete the table.

Length of side of square	1	2	3	4	5
Perimeter of square					
Area of square					

a. Are the length of a side of a square and the perimeter of the square related proportionally? Why or why not?

____

____

b. Are the length of a side of a square and the area of the square related proportionally? Why or why not?

____

____

16. Make a Conjecture A table shows a proportional relationship where k is the constant of proportionality. The rows are then switched. How does the new constant of proportionality relate to the original one?

____

LESSON 3.2

Rate of Change and Slope

COMMON CORE 8.F.4

...Determine the rate of change...of the function from...two (x, y) values, including reading these from a table or from a graph....

ESSENTIAL QUESTION

How do you find a rate of change or a slope?

Investigating Rates of Change

A **rate of change** is a ratio of the amount of change in the dependent variable, or *output*, to the amount of change in the independent variable, or *input*.

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EXAMPLE 1 Real World

COMMON CORE 8.F.4

Eve keeps a record of the number of lawns she has mowed and the money she has earned. Tell whether the rates of change are constant or variable.

	Day 1	Day 2	Day 3	Day 4
Number of lawns	1	3	6	8
Amount earned ($)	15	45	90	120

STEP 1 Identify the input and output variables.

Input variable: number of lawns Output variable: amount earned

STEP 2 Find the rates of change.

Day 1 to Day 2: $\frac{\text{change in \$}}{\text{change in lawns}} = \frac{45 - 15}{3 - 1} = \frac{30}{2} = 15$

Day 2 to Day 3: $\frac{\text{change in \$}}{\text{change in lawns}} = \frac{90 - 45}{6 - 3} = \frac{45}{3} = 15$

Day 3 to Day 4: $\frac{\text{change in \$}}{\text{change in lawns}} = \frac{120 - 90}{8 - 6} = \frac{30}{2} = 15$

The rates of change are constant: $15 per lawn.

Math Talk
Mathematical Practices

Would you expect the rates of change of a car's speed during a drive through a city to be constant or variable? Explain.

YOUR TURN

1. The table shows the approximate height of a football after it is kicked. Tell whether the rates of change are constant or variable.

Time (s)	Height (ft)
0	0
0.5	18
1.5	31
2	26

Find the rates of change:

____

The rates of change are **constant / variable.**

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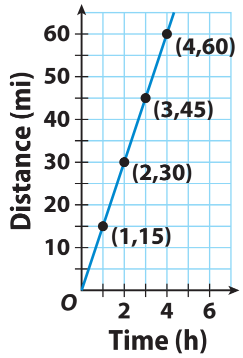
EXPLORE ACTIVITY Real World

COMMON CORE 8.F.4

Using Graphs to Find Rates of Change

You can also use a graph to find rates of change.

The graph shows the distance Nathan bicycled over time. What is Nathan's rate of change?



A Find the rate of change from 1 hour to 2 hours.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{30 - \square}{2 - 1} = \frac{\square}{1} = \square \text{ miles per hour}$$

B Find the rate of change from 1 hour to 4 hours.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{60 - \square}{4 - \square} = \frac{\square}{\square} = \square \text{ miles per hour}$$

C Find the rate of change from 2 hours to 4 hours.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{60 - \square}{4 - \square} = \frac{\square}{\square} = \square \text{ miles per hour}$$

D Recall that the graph of a proportional relationship is a line through the origin. Explain whether the relationship between Nathan's time and distance is a proportional relationship.

____

____

Reflect

2. **Make a Conjecture** Does a proportional relationship have a constant rate of change?

____

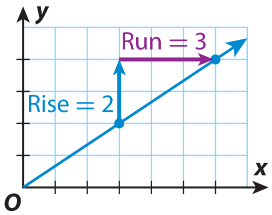
3. Does it matter what interval you use when you find the rate of change of a proportional relationship? Explain.

____

____

Calculating Slope *m*

When the rate of change of a relationship is constant, any segment of its graph has the same steepness. The constant rate of change is called the *slope* of the line.



Slope Formula

The **slope** of a line is the ratio of the change in *y*-values (rise) for a segment of the graph to the corresponding change in *x*-values (run).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

EXAMPLE 2

COMMON CORE 8.F.4

Find *m* the slope of the line.

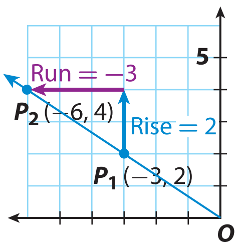
STEP 1 Choose two points on the line.

$P_1(x_1, y_1)$ $\qquad$ $P_2(x_2, y_2)$

STEP 2 Find the change in *y*-values (rise $= y_2 - y_1$) and the change in *x*-values (run $= x_2 - x_1$) as you move from one point to the other.

$$\text{rise} = y_2 - y_1 = 4 - 2 = 2$$

$$\text{run} = x_2 - x_1 = -6 - (-3) = -3$$



If you move up or right, the change is positive. If you move down or left, the change is negative.

STEP 3 $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2}{-3}$

$= -\frac{2}{3}$

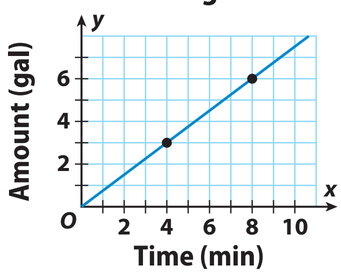
My Notes

YOUR TURN

4. The graph shows the rate at which water is leaking from a tank. The slope of the line gives the leaking rate in gallons per minute. Find the slope of the line.

Rise = ________ $\qquad$ Run = ________

Slope = ________



Guided Practice

Tell whether the rates of change are constant or variable. (Example 1)

1. building measurements ________________

Feet	3	12	27	75
Yards	1	4	9	25

2. computers sold ________________

Week	2	4	9	20
Number Sold	6	12	25	60

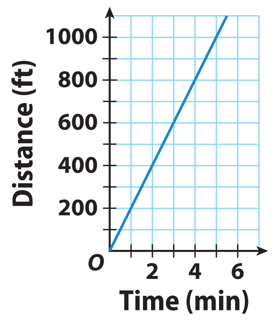
3. distance an object falls ________________

Distance (ft)	16	64	144	256
Time (s)	1	2	3	4

4. cost of sweaters ________________

Number	2	4	7	9
Cost ($)	38	76	133	171

Erica walks to her friend Philip's house. The graph shows Erica's distance from home over time. (Explore Activity)



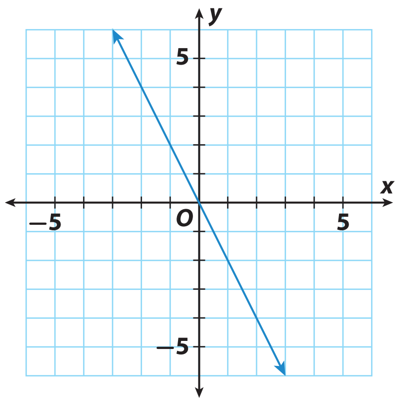
5. Find the rate of change from 1 minute to 2 minutes.

$$\frac{\text{change in distance}}{\text{change in time}} = \frac{400 - \square}{2 - \square} = \frac{\square}{\square} = \square \text{ ft per min}$$

6. Find the rate of change from 1 minute to 4 minutes. ________________

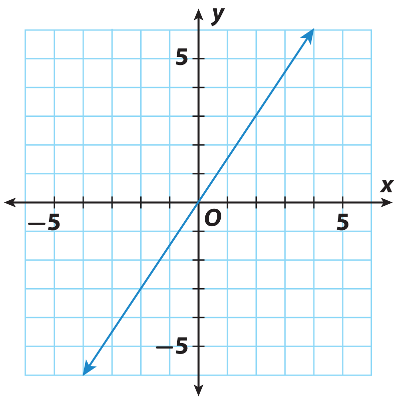
Find the slope of each line. (Example 2)

7.



slope = ________________

8.



slope = ________________

ESSENTIAL QUESTION CHECK-IN

9. If you know two points on a line, how can you find the rate of change of the variables being graphed?

____

____

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Name ______________________ Class ______________ Date ______________

3.2 Independent Practice

COMMON CORE 8.F.4

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10. Rectangle *EFGH* is graphed on a coordinate plane with vertices at $E(-3, 5)$, $F(6, 2)$, $G(4, -4)$, and $H(-5, -1)$.

a. Find the slopes of each side.

____

b. What do you notice about the slopes of opposite sides?

____

c. What do you notice about the slopes of adjacent sides?

____

11. A bicyclist started riding at 8:00 A.M. The diagram below shows the distance the bicyclist had traveled at different times. What was the bicyclist's average rate of speed in miles per hour?

____

12. **Multistep** A line passes through (6, 3), (8, 4), and $(n, -2)$. Find the value of n.

____

13. A large container holds 5 gallons of water. It begins leaking at a constant rate. After 10 minutes, the container has 3 gallons of water left.

a. At what rate is the water leaking?

____

b. After how many minutes will the container be empty?

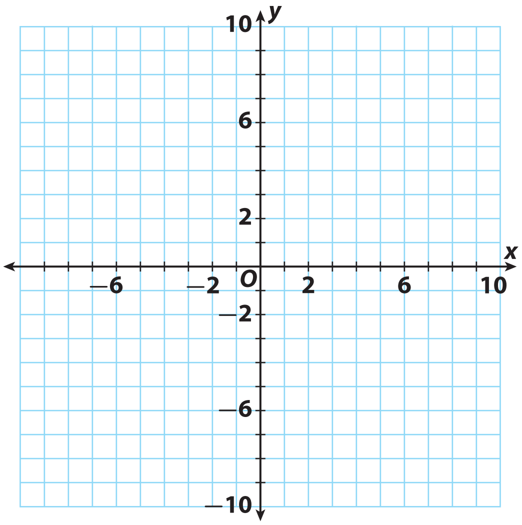
____

14. **Critique Reasoning** Billy found the slope of the line through the points (2, 5) and (−2, −5) using the equation $\frac{2-(-2)}{5-(-5)} = \frac{2}{5}$. What mistake did he make?

____

____

15. **Multiple Representations** Graph parallelogram *ABCD* on a coordinate plane with vertices at $A(3, 4)$, $B(6, 1)$, $C(0, -2)$, and $D(-3, 1)$.



a. Find the slope of each side.

____

____

b. What do you notice about the slopes?

____

____

c. Draw another parallelogram on the coordinate plane. Do the slopes have the same characteristics?

____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

16. **Communicate Mathematical Ideas** Ben and Phoebe are finding the slope of a line. Ben chose two points on the line and used them to find the slope. Phoebe used two different points to find the slope. Did they get the same answer? Explain.

____

____

____

17. **Analyze Relationships** Two lines pass through the origin. The lines have slopes that are opposites. Compare and contrast the lines.

____

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____

____

____

18. **Reason Abstractly** What is the slope of the *x*-axis? Explain.

____

____

____

LESSON **3.3**

Interpreting the Unit Rate as Slope

COMMON CORE 8.EE.5

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *Also 8.F.2, 8.F.4*

ESSENTIAL QUESTION

How do you interpret the unit rate as slope?

EXPLORE ACTIVITY Real World

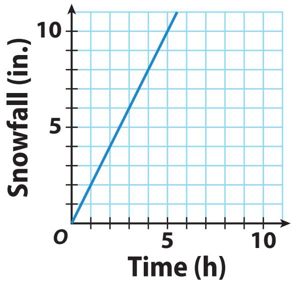
COMMON CORE 8.EE.5, 8.F.4

Relating the Unit Rate to Slope

A rate is a comparison of two quantities that have different units, such as miles and hours. A **unit rate** is a rate in which the second quantity in the comparison is one unit.

A storm is raging on Misty Mountain. The graph shows the constant rate of change of the snow level on the mountain.

Misty Mountain Storm



A Find the slope of the graph using the points (1, 2) and (5, 10). Remember that the slope is the constant rate of change.

B Find the unit rate of snowfall in inches per hour. Explain your method.

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C Compare the slope of the graph and the unit rate of change in the snow level. What do you notice?

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D Which unique point on this graph can represent the slope of the graph and the unit rate of change in the snow level? Explain how you found the point.

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Graphing Proportional Relationships

You can use a table and a graph to find the unit rate and slope that describe a real-world proportional relationship. The constant of proportionality for a proportional relationship is the same as the slope.

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EXAMPLE 1 Real World

COMMON CORE 8.EE.5

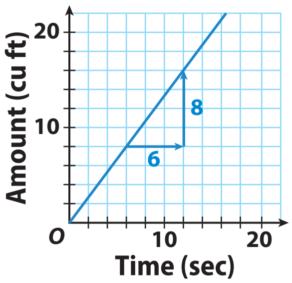
Every 3 seconds, 4 cubic feet of water pass over a dam. Draw a graph of the situation. Find the unit rate of this proportional relationship.

STEP 1 Make a table.

Time (s)	3	6	9	12	15
Volume (ft^3)	4	8	12	16	20

STEP 2 Draw a graph.

Water Over the Dam



STEP 3 Find the slope.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{8}{6}$$

$$= \frac{4}{3}$$

The unit rate of water passing over the dam and the slope of the graph of the relationship are equal, $\frac{4}{3}$ cubic feet per second.

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Math Talk
Mathematical Practices
In a proportional relationship, how are the constant of proportionality, the unit rate, and the slope of the graph of the relationship related?

Reflect

1. **What If?** Without referring to the graph, how do you know that the point $\left(1, \frac{4}{3}\right)$ is on the graph?

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YOUR TURN

2. Tomas rides his bike at a steady rate of 2 miles every 10 minutes. Graph the situation. Find the unit rate of this proportional relationship.

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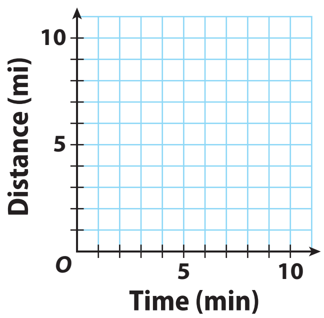
Tomas's Ride



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Using Slopes to Compare Unit Rates

You can compare proportional relationships presented in different ways.

EXAMPLE 2 Real World

COMMON CORE 8.EE.5, 8.F.2

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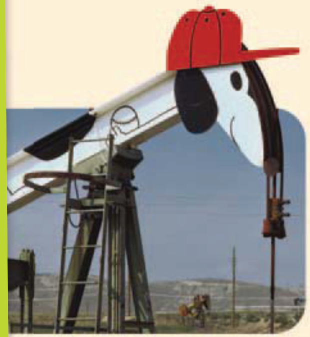
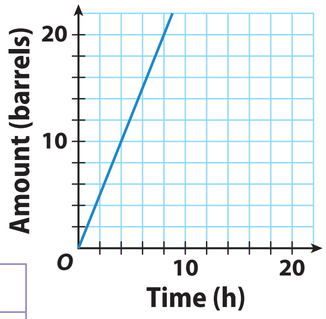


The equation $y = 2.75x$ represents the rate, in barrels per hour, that oil is pumped from Well A. The graph represents the rate that oil is pumped from Well B. Which well pumped oil at a faster rate?

Well B Pumping Rate



STEP 1 Use the equation $y = 2.75x$ to make a table for Well A's pumping rate, in barrels per hour.

Time (h)	1	2	3	4
Quantity (barrels)	2.75	5.5	8.25	11

STEP 2 Use the table to find the slope of the graph of Well A.

slope = unit rate $= \frac{5.5 - 2.75}{2 - 1} = \frac{2.75}{1} =$ **2.75** barrels/hour

STEP 3 Use the graph to find the slope of the graph of Well B.

slope = unit rate $= \frac{\text{rise}}{\text{run}} = \frac{10}{4} =$ **2.5** barrels/hour

STEP 4 Compare the unit rates.

$2.75 > 2.5$, so Well A's rate, 2.75 barrels/hour, is faster.

Reflect

3. Describe the relationships among the slope of the graph of Well A's rate, the equation representing Well A's rate, and the constant of proportionality.

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YOUR TURN

4. The equation $y = 375x$ represents the relationship between x, the time that a plane flies in hours, and y, the distance the plane flies in miles for Plane A. The table represents the relationship for Plane B. Find the slope of the graph for each plane and the plane's rate of speed. Determine which plane is flying at a faster rate of speed.

Time (h)	1	2	3	4
Distance (mi)	425	850	1275	1700

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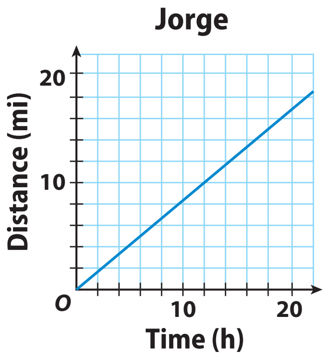
Guided Practice

Give the slope of the graph and the unit rate. (Explore Activity and Example 1)

1. Jorge: 5 miles every 6 hours



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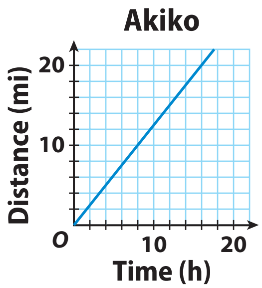
2. Akiko

Time (h)	4	8	12	16
Distance (mi)	5	10	15	20



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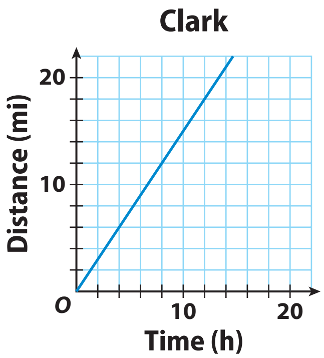
3. The equation $y = 0.5x$ represents the distance Henry hikes, in miles, over time, in hours. The graph represents the rate that Clark hikes. Determine which hiker is faster. Explain. (Example 2)



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Write an equation relating the variables in each table. (Example 2)

4.

Time (*x*)	1	2	4	6
Distance (*y*)	15	30	60	90

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5.

Time (*x*)	16	32	48	64
Distance (*y*)	6	12	18	24

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ESSENTIAL QUESTION CHECK-IN

6. Describe methods you can use to show a proportional relationship between two variables, *x* and *y*. For each method, explain how you can find the unit rate and the slope.

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Name ____________________ Class ____________ Date ____________

3.3 Independent Practice

COMMON CORE 8.EE.5, 8.F.2, 8.F.4

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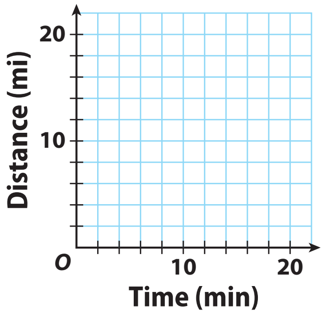
7. A Canadian goose migrated at a steady rate of 3 miles every 4 minutes.

a. Fill in the table to describe the relationship.

Time (min)	4	8			20
Distance (mi)			9	12	

b. Graph the relationship.

Migration Flight



c. Find the slope of the graph and describe what it means in the context of this problem.

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8. **Vocabulary** A unit rate is a rate in which the

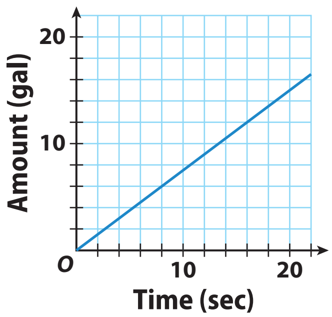
first quantity / second quantity in the comparison is one unit.

9. The table and the graph represent the rate at which two machines are bottling milk in gallons per second.

Machine 1

Time (s)	1	2	3	4
Amount (gal)	0.6	1.2	1.8	2.4

Machine 2



a. Determine the slope and unit rate of each machine.

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b. Determine which machine is working at a faster rate.

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10. **Cycling** The equation $y = \frac{1}{9}x$ represents the distance y, in kilometers, that Patrick traveled in x minutes while training for the cycling portion of a triathlon. The table shows the distance y Jennifer traveled in x minutes in her training. Who has the faster training rate?

Time (min)	40	64	80	96
Distance (km)	5	8	10	12

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

11. **Analyze Relationships** There is a proportional relationship between minutes and dollars per minute, shown on a graph of printing expenses. The graph passes through the point (1, 4.75). What is the slope of the graph? What is the unit rate? Explain.

12. **Draw Conclusions** Two cars start at the same time and travel at different constant rates. A graph for Car A passes through the point (0.5, 27.5), and a graph for Car B passes through (4, 240). Both graphs show distance in miles and time in hours. Which car is traveling faster? Explain.

13. **Critical Thinking** The table shows the rate at which water is being pumped into a swimming pool.

Time (min)	2	5	7	12
Amount (gal)	36	90	126	216

Use the unit rate and the amount of water pumped after 12 minutes to find how much water will have been pumped into the pool after $13\frac{1}{2}$ minutes. Explain your reasoning.

Ready to Go On?

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3.1 Representing Proportional Relationships

1. Find the constant of proportionality for the table of values.

x	2	3	4	5
y	3	4.5	6	7.5

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2. Phil is riding his bike. He rides 25 miles in 2 hours, 37.5 miles in 3 hours, and 50 miles in 4 hours. Find the constant of proportionality and write an equation to describe the situation.

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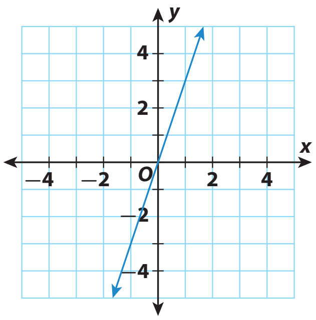
3.2 Rate of Change and Slope

Find the slope of each line.

3.



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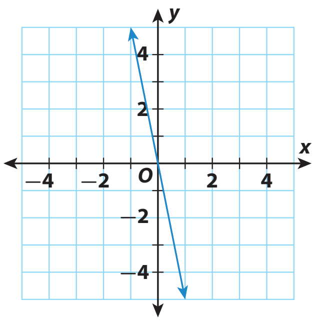
4.



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3.3 Interpreting the Unit Rate as Slope

5. The distance Train A travels is represented by $d = 70t$, where d is the distance in kilometers and t is the time in hours. The distance Train B travels at various times is shown in the table. What is the unit rate of each train? Which train is going faster?

Time (hours)	Distance (km)
2	150
4	300
5	375

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ESSENTIAL QUESTION

6. What is the relationship among proportional relationships, lines, rates of change, and slope?

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COMMON CORE

MODULE 3 **MIXED REVIEW**

Assessment Readiness

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Selected Response

1. Which of the following is equivalent to 5^{-1}?

Ⓐ 4

Ⓑ $\frac{1}{5}$

Ⓒ $-\frac{1}{5}$

Ⓓ -5

2. Prasert earns $9 an hour. Which table represents this proportional relationship?

Ⓐ

Hours	4	6	8
Earnings ($)	36	54	72

Ⓑ

Hours	4	6	8
Earnings ($)	36	45	54

Ⓒ

Hours	2	3	4
Earnings ($)	9	18	27

Ⓓ

Hours	2	3	4
Earnings ($)	18	27	54

3. A factory produces widgets at a constant rate. After 4 hours, 3,120 widgets have been produced. At what rate are the widgets being produced?

Ⓐ 630 widgets per hour

Ⓑ 708 widgets per hour

Ⓒ 780 widgets per hour

Ⓓ 1,365 widgets per hour

4. A full lake begins dropping at a constant rate. After 4 weeks it has dropped 3 feet. What is the unit rate of change in the lake's level compared to its full level?

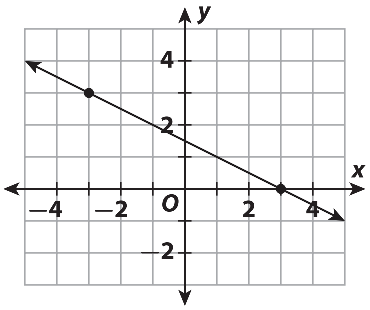
Ⓐ 0.75 feet per week

Ⓑ 1.33 feet per week

Ⓒ −0.75 feet per week

Ⓓ −1.33 feet per week

5. What is the slope of the line below?



Ⓐ -2

Ⓑ $-\frac{1}{2}$

Ⓒ $\frac{1}{2}$

Ⓓ 2

6. Jim earns $41.25 in 5 hours. Susan earns $30.00 in 4 hours. Pierre's hourly rate is less than Jim's, but more than Susan's. What is his hourly rate?

Ⓐ $6.50

Ⓑ $7.75

Ⓒ $7.35

Ⓓ $8.25

Mini-Task

7. Joelle can read 3 pages in 4 minutes, 4.5 pages in 6 minutes, and 6 pages in 8 minutes.

a. Make a table of the data.

Minutes			
Pages			

b. Use the values in the table to find the unit rate.

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c. Graph the relationship between minutes and pages read.