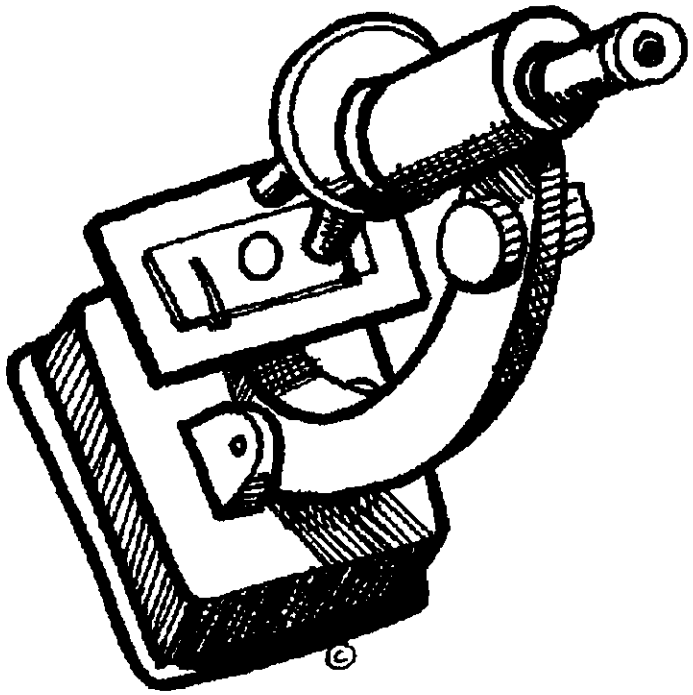
Highlands High School

Science Department Reference Manual



Second Edition ~ Version 2

Scientific Method

1. Identify a problem/question/purpose (must be in sentence form)

2. Gather background information
   - What do I already know?
   - Key words/vocabulary
   - Identify variables/units of measurement

3. Create a hypothesis (must be in sentence form)
   - Must be a positive statement
   - Ex) I predict the soil will increase in temperature.

4. Test hypothesis/design experiment/follow procedure

5. Observe and collect/record data
   - Graphs, charts, time

6. Analyze data/reflect on results

7. Conclusion
   - Restate the question
   - Identify and use key words
   - Incorporate data from lab to support ideas
   - Prove or disprove hypothesis

All grade levels will follow these steps. In 6th grade, the conclusion may be a set of specific questions about the lab. In 7th grade, part of the conclusion will be completed. In 8th grade, all of the conclusion will be completed.

FORMAL LABORATORY REPORTS

Formal labs will be done on the computer/typed (word processor, spreadsheets and graphs). Follow the format below.

TITLE:

OBJECTIVE: The purpose of the experiment should answer the questions "What and Why?"

MATERIALS: List any and all materials/equipment used in the lab.

THEORY: This should be a discussion that must include pertinent vocabulary, definitions, equations, formulas and constants used. The discussion should also contain information about significant individuals relevant to the lab, if applicable.

PROCEDURE: In detailed paragraph form, you should describe the necessary steps to complete the lab. By following your procedure, the lab should be reproducible by anyone.

DATA: Include a copy of all data collected during the experiment in typed table form. Data must include measurements and observations obtained from the procedure.

CALCULATIONS:
Show all results of calculations involving data and error analysis. One example of each type of calculation is sufficient.

ERROR ANALYSIS FORM:

$$\% \text{ error} = \frac{\text{abs [theoretical value} - \text{experimental value]}}{\text{theoretical value}} \times 100\%$$

$$\text{relative error} = \frac{\text{abs [difference between two values]}}{\text{average of two values}} \times 100\%$$

GRAPHS: Follow the graphing protocol, when applicable.

DISCUSSION: This section should be in essay form. Discussion points should include answers to questions, relationship of data and theory, an account of errors, and final summation of ideas. This discussion essay must be evidence-based.

RAW DATA: Be sure to attach your raw data to the end of the formal lab report.
(Pre-Lab)

Highlands High School
Science Department Graphing Protocol

Guidelines for Creating a Graph

General:

- Make use of the entire graph paper
- Use a ruler for all "straight" lines

Title:

- **Y** Label vs. **X** Label

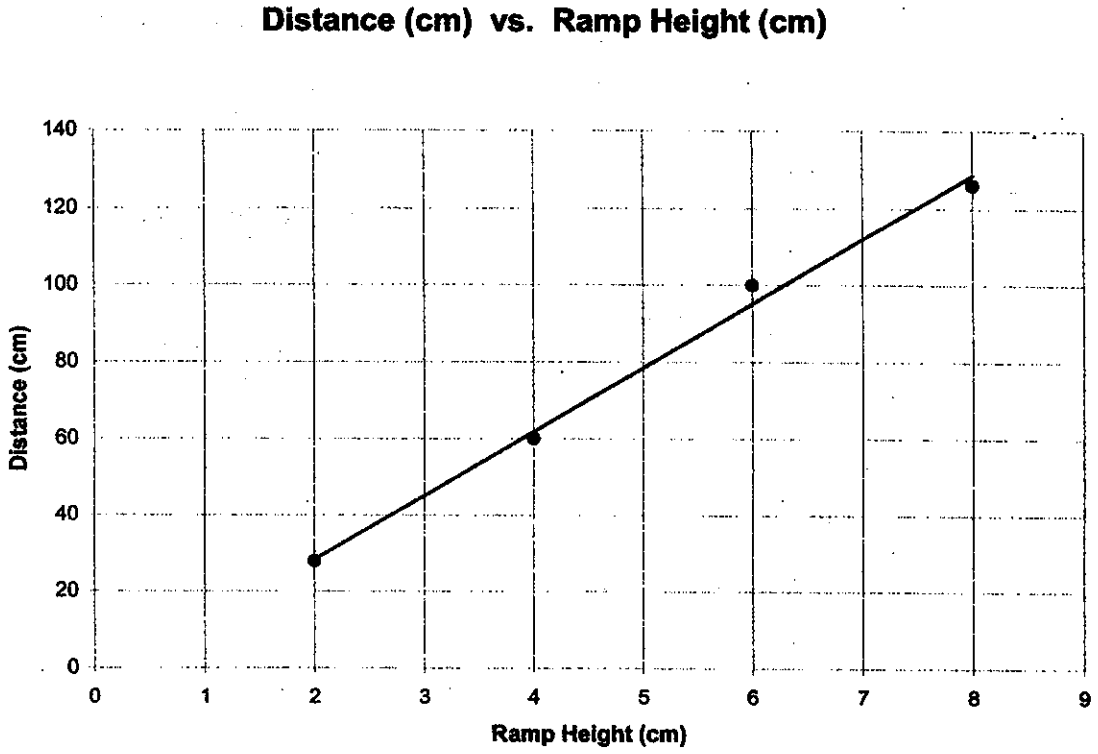
Axes:

- Labels
- Units
- Consistent scale
- Assign X axis as the ***control variable***
- Assign Y axis as the ***dependent variable***

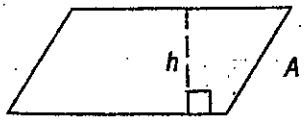
Analysis and Interpretation:

- Best Fit Line
- Equation with correlation
- Relationship between data and concept

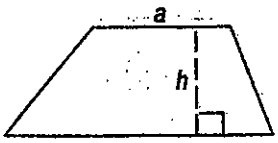
Sample Graph



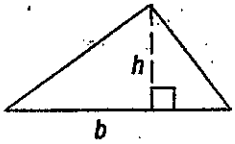
$P = 2l + 2w$
$A = lw$



$A = bh$

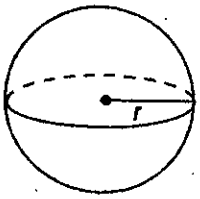


$A = \frac{h}{2}(a + b)$

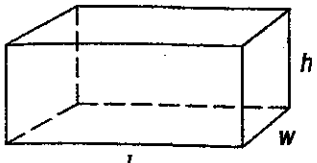


$A = \frac{1}{2}bh$

$C = 2\pi r$
$A = \pi r^2$

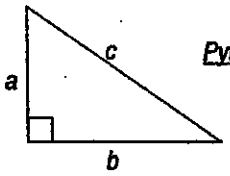


$SA = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$



$SA = 2lw + 2lh + 2wh$
$V = lwh$

$SA = 2\pi r^2 + 2\pi rh$
$V = \pi r^2 h$



<u>Pythagorean Theorem</u>:
$c^2 = a^2 + b^2$

<u>Law of Sines</u>: $\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$

<u>Law of Cosines</u>: $c^2 = a^2 + b^2 - 2ab \cos \angle C$

<u>Simple Interest</u>: $I = Prt$

<u>Compound Interest (n times per year)</u>: $A = P(1 + \frac{r}{n})^{nt}$

<u>Quadratic Formula</u>:

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

<u>Distance Formula</u>:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

<u>Trigonometric Ratios</u>:

$\sin \theta = \frac{opp}{hyp}$ $\cos \theta = \frac{adj}{hyp}$

$\tan \theta = \frac{opp}{adj}$

<u>Permutations</u>: $P(n, r) = \frac{n!}{(n - r)!}$

<u>Combinations</u>: $C(n, r) = \frac{n!}{r!(n - r)!}$

<u>n^{th} Term of an Arithmetic Sequence</u>: $a_n = a + (n - 1)d$

<u>n^{th} Term of a Geometric Sequence</u>: $a_n = ar^{(n-1)}$

<u>Sum of an Arithmetic Series</u>:

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

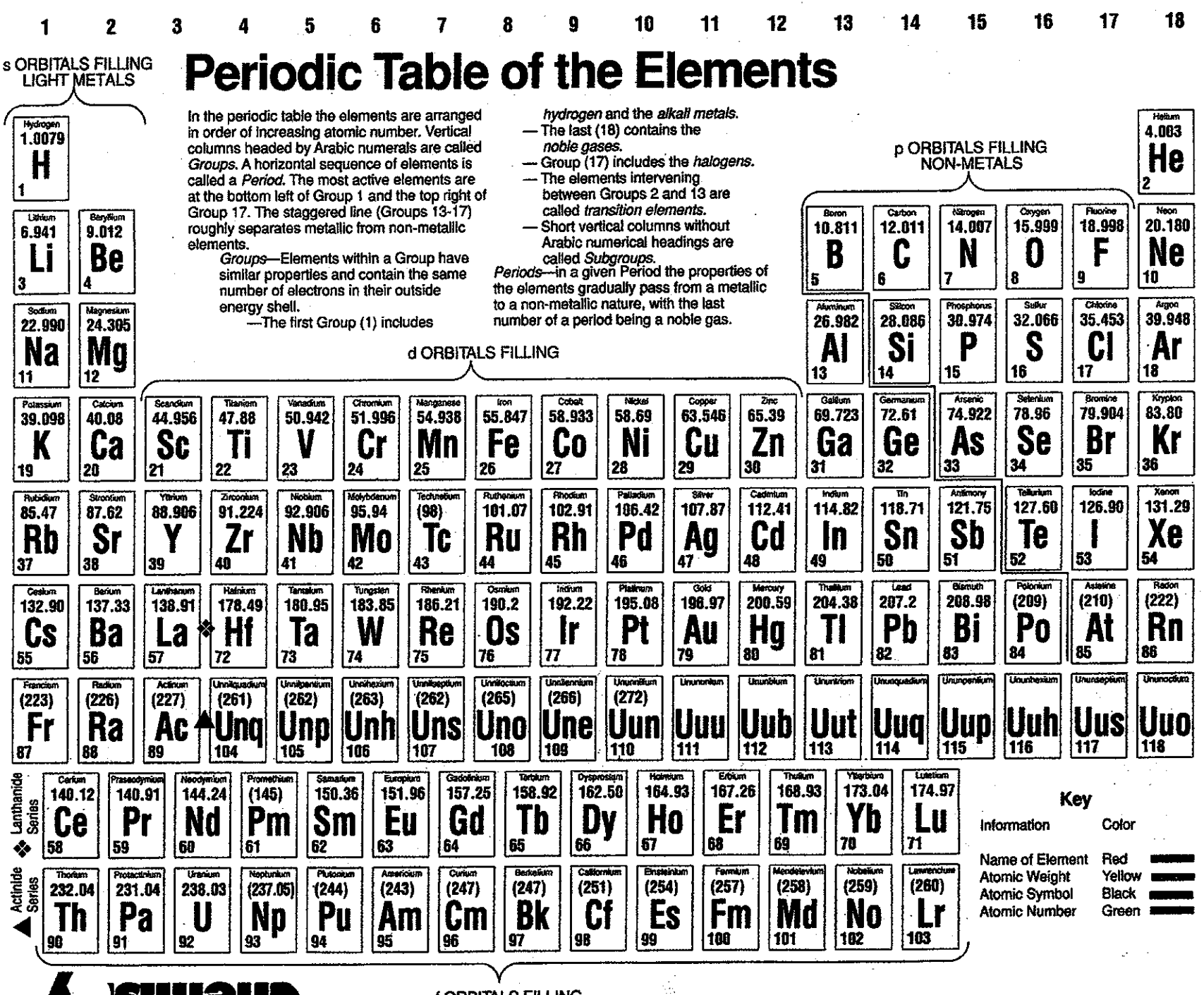
<u>Sum of a Geometric Series</u>:

$S_n = \frac{a - ar^n}{1 - r}$ or $S_n = \frac{a(1 - r^n)}{1 - r}$

<u>Sum of an Infinite Geometric Series</u>: $S = \frac{a}{1 - r}$

<u>Base Change Formula</u>: $\log_b x = \frac{\log_c x}{\log_c b}$

Periodic Table of the Elements



In the periodic table the elements are arranged in order of increasing atomic number. Vertical columns headed by Arabic numerals are called *Groups*. A horizontal sequence of elements is called a *Period*. The most active elements are at the bottom left of Group 1 and the top right of Group 17. The staggered line (Groups 13-17) roughly separates metallic from non-metallic elements.

Groups—Elements within a Group have similar properties and contain the same number of electrons in their outside energy shell.

—The first Group (1) includes *hydrogen* and the *alkali metals*.

—The last (18) contains the *noble gases*.

— Group (17) includes the *halogens*.

— The elements intervening between Groups 2 and 13 are called *transition elements*.

— Short vertical columns without Arabic numerical headings are called *Subgroups*.

Periods—in a given Period the properties of the elements gradually pass from a metallic to a non-metallic nature, with the last number of a period being a noble gas.

Key

Information	Color
Name of Element	Red
Atomic Weight	Yellow
Atomic Symbol	Black
Atomic Number	Green

Chemistry

METRIC CONVERSION FACTORS (Approximate)

Conversions TO Metric Measures

LENGTH

Symbol	When You Know	Multiply by	To Find	Symbol
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km

AREA

Symbol	When You Know	Multiply by	To Find	Symbol
in^2	square miles	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha

MASS (weight)

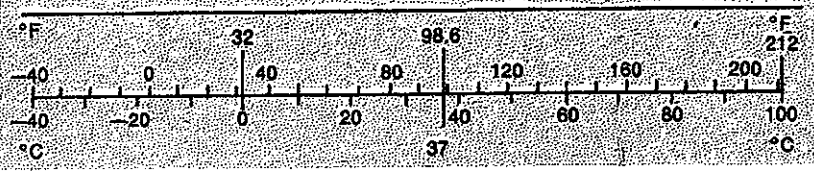
Symbol	When You Know	Multiply by	To Find	Symbol
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t

VOLUME

Symbol	When You Know	Multiply by	To Find	Symbol
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3

TEMPERATURE (exact)

Symbol	When You Know	Multiply by	To Find	Symbol
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C



*1 in = 2.54 cm (exactly).

Conversions FROM Metric Measures

LENGTH

Symbol	When You Know	Multiply by	To Find	Symbol
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi

AREA

Symbol	When You Know	Multiply by	To Find	Symbol
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	

MASS (weight)

Symbol	When You Know	Multiply by	To Find	Symbol
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	

VOLUME

Symbol	When You Know	Multiply by	To Find	Symbol
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3

TEMPERATURE (exact)

Symbol	When You Know	Multiply by	To Find	Symbol
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F

METRIC CONVERSION FACTORS • Courtesy U.S. Dept. of Commerce • National Bureau of Standards

BASIC LAWS and CONVERSIONS

CONVERSIONS

1 kilometer = 1000 meters
1 decimeter = 1/10 meter
1 centimeter = 1/100 meter
1 millimeter = 1/1000 meter
1 liter = 1000 milliliters
1 inch = 2.54 centimeters
1 foot = 30.5 centimeters
1 yard = 0.92 meters
1 pound = 453.6 grams
1 kilogram = 1000 grams, or 2.2 pounds

HEAT

To convert Fahrenheit to Centigrade: Subtract 32 from F, then multiply by 5/9, written C = 5/9 (F − 32). Note—Centigrade is now referred to as Celsius. (Note: 212F = 100C). To convert Centigrade to Fahrenheit: Multiply C by 9/5, then add 32, written F = (9/5C) + 32.

To convert Centigrade to Absolute or Kelvin scale, add 273 to C.

To convert Fahrenheit to Absolute or Kelvin scale, first convert F to C, then add 273.

Boyle's Law: $p_1 \times V_1 = p_2 \times V_2$ at constant temperature. Zero degrees Kelvin is the lowest possible temp.

In Kelvin Absolute temperature scale: water boils at 373K, freezes at 273K.

Charles' Law: $\frac{V_1}{V_2} = \frac{T_1}{T_2}$ **at const. press.**

Combination of Charles' and Boyle's Laws:

$$\frac{V_1 p_1}{T_1} = \frac{V_2 p_2}{T_2}$$

When heated through one degree Centigrade, any gas expands $\frac{1}{273}$ of its volume at 0 degrees Centigrade if the pressure remains constant. One **BTU** is the heat required to raise the temperature of 1 pound of water through 1 degree Fahrenheit.

One calorie is the heat required to raise the temperature of 1 gram of water through 1 degree Centigrade.

Specific Heat: heats required to raise the temperature of a unit mass of that substance through 1 degree. If H is total heat and M is mass, $H = M \times s \times (t_2 - t_1)$.

Heat of melting, or heat of fusion, L, is the quantity of heat needed to melt one unit weight of substance without changing its temperature, or $H = M \times L$.

80 calories of heat is required to melt 1 gram of ice without raising its temperature.

Boiling point of liquid: that temperature at which the vapor pressure is equal to the pressure above the liquid.

0.427 kilogram-meter (kg-m) = 1 calorie

$$\frac{\text{Work}}{\text{mechanical equivalent of heat}}$$

The work required to produce a unit quantity of heat is called the mechanical equivalent of heat.

HORSEPOWER

1 horsepower = 550 ft-lb/sec

$$\textbf{Horsepower} = \frac{\text{force (lb)} \times \text{distance (ft)}}{\text{550-ft-lb/sec} \times \text{time (sec)}}$$

$$\textbf{Friction constant} = \frac{\text{friction force}}{\text{weight}}$$

Work = Force × Distance moved

$$\textbf{Power} = \frac{\text{work}}{\text{time}}$$

1 watt = 10,200 gram-centimeters per sec.
1 kilowatt is 1000 watts
1 kilowatt is approximately 1⅓ horsepower.
Dyne is absolute metric unit of force. **Erg** is its unit of work.
1 Erg = force of 1 dyne acting through 1 centimeter.
1 Joule = 10,000,000 ergs, or about ¾ foot pounds.
The law of work when friction is neglected: Effort force × effort distance = resistance force × resistance distance.

Mechanical advantage of a machine =

$$\frac{\text{resistance force}}{\text{effort force}}$$

When friction is zero, mechanical advantage of a machine =

$$\frac{\text{effort distance}}{\text{resistance distance}}$$

Mechanical advantage of a lever =

$$\frac{\text{effort arm}}{\text{resistance arm}}$$

Moment of force = force × lever arm

Frictionless mechanical advantage of an inclined plane =

$$\frac{\text{length}}{\text{height}}$$

Frictionless mechanical advantage of a wheel and axle =

$$\frac{\text{circumference of wheel}}{\text{circumference of axle}}$$

PRESSURES AND DENSITIES

$$\textbf{Pressure} = \frac{\text{force}}{\text{area}}$$

1 column of water 1 foot deep = 62.4 pounds per square foot, or 0.433 pounds per square inch. 1 column of water 1 centimeter deep = 1 gram per square centimeter.

Specific gravity = number of times a substance is as heavy as an equal body of water, or Specific gravity (liquid) =

$$\frac{\text{weight of liquid}}{\text{weight of equal volume of water}}$$

$$\textbf{Density} = \frac{\text{mass}}{\text{volume}}$$

Pressure = depth × density, or force per unit area. An increase in pressure is transmitted equally through the liquid.

Specific gravity (solid) =

$$\frac{\text{weight of body}}{\text{weight of equal volume of water}}$$

or Specific gravity (solid) =

$$\frac{\text{weight of body}}{\text{loss of weight in water}}$$

One cubic yard of air weighs about 2 pounds. Atmospheric pressure at sea level = about 15 pounds per square inch.

VELOCITIES AND ENERGIES

$$\textbf{Velocity} = \frac{\text{distance}}{\text{time}}$$

$$\textbf{Acceleration} = \frac{\text{change of velocity}}{\text{time}}$$

$$\textbf{Acceleration of gravity} = \frac{32 \text{ ft/sec}}{\text{sec}}$$

$$\textbf{Centripetal force} = \frac{\text{weight}}{\text{acceleration of gravity}} \times \frac{(\text{velocity})^2}{\text{radius}}$$

Potential energy = weight of body × elevation

$$\textbf{Kinetic energy} = \frac{1}{2} \frac{\text{weight}}{\text{acceleration of gravity}} \times (\text{velocity})^2$$

Momentum = mass of body × its velocity

$$\textbf{Mass} = \frac{\text{weight}}{\text{acceleration of gravity}}$$

$$\textbf{Period of a pendulum: } T = 2\pi \sqrt{\frac{L}{g}}$$

Wave velocity = wave frequency × wave length, or $v = n \times l$

Speed of sound: 1090 feet per second in air at 0 degrees Centigrade. Velocity of sound increases 2 feet per second for each degree-Centigrade rise in temperature above zero degrees Centigrade.

ELECTRICITY

1 ampere = 1 coulomb per second
1 volt = 1 joule per coulomb

$$\textbf{Ohm's Law: Current} = \frac{\text{potential difference}}{\text{resistance}}$$

$$\text{or } \textbf{amperes} = \frac{\text{volts}}{\text{ohms}} \text{ or } I = \frac{V}{R}$$

Ampere = electric current; **volt** = potential difference; **ohm** = electrical resistance.

One volt potential difference will drive 1 ampere through a resistance of 1 ohm.

The resistance of a conductor can be calculated by the formula:

$$R = \frac{kl}{d^2}$$ (where l is length, d is diameter, and k is constant.)

The combined resistance of conductors connected in parallel is

$$\frac{1}{R_c} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

1 watt is the power of a current on 1 ampere when the potential difference is 1 volt.

To compute electric power: P (power in watts) = V (voltage in volts) × I (current in amperes), or $P = V \times I$.

To compute the heat (H), produced by a current (I), through a resistance (R), in a time (t), use the equation:

$$H = I^2 \times R \times t \times 0.24 \text{ cal/watt-sec.}$$

LIGHT AND LENSES

1 foot-candle: The illumination of any point on a surface 1 foot from a standard candle.

$$\textbf{Illumination (ft-c)} = \frac{\text{intensity (candles)}}{\text{distance in feet}^2}$$

Velocity of light = 186,000 miles per sec.

Index of refraction =

$$\frac{\text{velocity of light in air}}{\text{velocity of light in the substance}}$$

Lens image equation: $\frac{1}{D_o} \times \frac{1}{D_i} = \frac{1}{f}$

Magnification =

$$\frac{\text{image length}}{\text{object length}} \text{ or } \frac{\text{image distance}}{\text{object distance}}$$

PHYSICS FORMULAS

SCIENTIFIC NOTATION

Prefixes and symbols to form decimal multiples and/or submultiples.

Power of ten	E Notation	Decimal Equivalent	Prefix	Phonic	Symbol
10^{12}	E + 12	1 000 000 000 000	tera	ter'a	T
10^{9}	E + 09	1 000 000 000	giga	ji'ga	G
10^{6}	E + 06	1 000 000	mega	meg'a	M
10^{3}	E + 03	1 000	kilo	kil'o	k
10^{2}	E + 02	100	hecto	hek'to	h
10	E + 01	10	deka	dek'a	da
10^{-1}	E - 01	0.1	deci	des'i	d
10^{-2}	E - 02	0.01	centi	sen'ti	c
10^{-3}	E - 03	0.001	milli	mil'i	m
10^{-6}	E - 06	0.000 001	micro	mi'kro	μ
10^{-9}	E - 09	0.000 000 001	nano	nan'o	n
10^{-12}	E - 12	0.000 000 000 001	pico	pe'ko	p
10^{-15}	E - 15	0.000 000 000 000 001	femto	fem'to	f
10^{-18}	E - 18	0.000 000 000 000 000 001	atto	at'to	a

KINEMATIC FORMULAS

Coefficient of Friction: $\mu = \frac{F}{N}$

μ = coefficient of friction
F = force of friction N = force normal to surface

Velocity: $v_{av} = \frac{d}{t}$

v_{av} = average velocity
d = distance t = elapsed time

Acceleration: $a_{av} = \frac{v_f - v_i}{t}$

a_{av} = average acceleration v_i = initial velocity
v_f = final velocity t = elapsed time

Newton's 2nd Law of Motion: $F = m \cdot a$
F = force m = mass a = acceleration

Law of Universal Gravitation: $F = G\frac{m_1 \cdot m_2}{d^2}$

F = force of attraction $m_1 \cdot m_2$ = product of masses
G = gravitational constant d = distance between their centers

Centripetal Force: $F = \frac{m \cdot v^2}{r}$

F = cetripetal force
m = mass v = velocity r = radius of path

Pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$

T = period l = length g = acceleration of gravity

Work: $W = F \cdot d$
W = work F = force d = distance

Ideal Mechanical Advantage: IMA = d_A/d_L
d_A = distance through which applied force F acts
d_L = distance load moves against force W without friction
Actual Mechanical Advantage: AMA = W/F

Mechanical Equivalent of Heat: $W = J \cdot Q$
W = work J = mechanical equivalent of heat Q = heat

ENERGY RELATIONSHIPS

Kinetic Energy: $K = \frac{1}{2} m \cdot v^2$
K = kinetic energy m = mass v = velocity

Potential Energy: $V = m \cdot g \cdot h$
V = potential g = acceleration of gravity
W = mass h = vertical distance (height)

Relationship between Mass and Energy: $E = m \cdot c^2$
E = energy m = mass c = velocity of light

OPTICAL RELATIONSHIPS

Wave Formula: $v = f \cdot \lambda$
v = wave speed f = frequency λ = wave length

Uniformly Illuminated Surface: $E = \frac{lm}{A}$
E = illumination
lm = luminous flux A = uniformly illuminated area

Images in Mirrors and Lenses: $\frac{S_o}{S_i} = \frac{D_o}{D_i}$

S_o = object size D_o = object distance
S_i = image size D_i = image distance

Focal Length of Mirrors and Lenses: $\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i}$
f = focal length
D_o = object distance D_i = image distance

Snell's Law: $n_i \sin \Theta_i = n_r \sin \Theta_r$
n_i = refractive index of ith material
Θ_i = angle between ray and normal to surface

ELECTRICITY AND MAGNETISM

Electric Current: $I = \frac{q}{t}$

I = current q = quantity of charge t = time

Coulomb's Law of Electrostatics: $F = k\frac{q_1 \cdot q_2}{d^2}$
F = force between two charges
k = proportionality constant
$q_1 \cdot q_2$ = product of charges
d = distance separating charges

Capacitance of a Capacitor: $C = \frac{q}{V}$
C = capacitance of a capacitor
V = potential difference between plates
q = charge on either plate

Ohm's Law of Resistance: $E = I \cdot R$
E = emf of source I = current in the circuit
R = resistance of the circuit

Joule's Law: $Q = I^2 \cdot R \cdot t$
Q = heat energy I = current R = resistance t = time

Faraday's Law of Electrolysis: $m = z \cdot I \cdot t$
m = mass z = electrochemical equivalent
I = current t = time

Induced emf: Coil in a Magnetic Field: $E = -N\frac{\Delta\Phi}{\Delta t}$
E = induced emf N = number of turns
$\Delta\Phi/\Delta t$ = the change in flux linkage in a given interval of time

Induced emf: Conductor in a Magnetic Field: $E = B \cdot l \cdot v$
E = induced emf B = flux density of the magnetic field
l = length of conductor
v = velocity of conductor across magnetic field

Instantaneous Voltage: $e = E_{max} \sin \Theta$
e = instantaneous voltage
E_{max} = maximum voltage
Θ = angle between the plane of the conducting loop and the perpendicular to the magnetic flux (displacement angle)

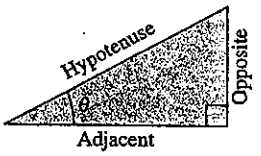
Instantaneous Current: $i = I_{max} \sin \Theta$
i = instantaneous current
I_{max} = maximum current
Θ = displacement angle

Math/Physics

TRIGONOMETRY

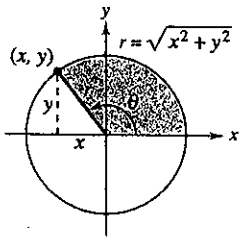
Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.

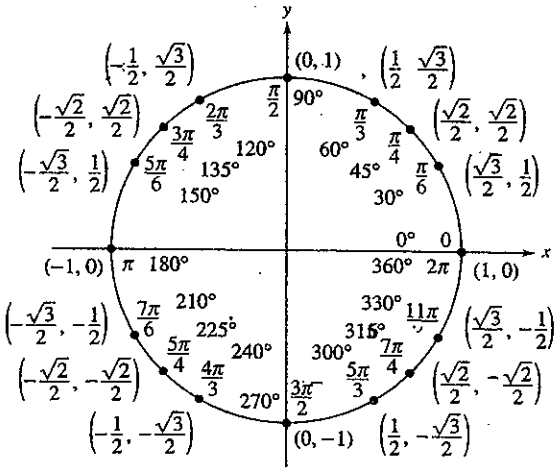


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Circular function definitions, where θ is any angle.



$$\sin\theta = \frac{y}{r} \quad \csc\theta = \frac{r}{y}$$
$$\cos\theta = \frac{x}{r} \quad \sec\theta = \frac{r}{x}$$
$$\tan\theta = \frac{y}{x} \quad \cot\theta = \frac{x}{y}$$



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \sec x = \frac{1}{\cos x} \quad \tan x = \frac{1}{\cot x}$$
$$\csc x = \frac{1}{\sin x} \quad \cos x = \frac{1}{\sec x} \quad \cot x = \frac{1}{\tan x}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$
$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \quad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$
$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Reduction Formulas

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x$$
$$\csc(-x) = -\csc x \quad \tan(-x) = -\tan x$$
$$\sec(-x) = \sec x \quad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$
$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$
$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$
$$\cos^2 u = \frac{1 + \cos 2u}{2}$$
$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$
$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$
$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$
$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$
$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$